Test 3 - MTH 1310

Dr. Graham-Squire, Summer 2012

9:43
10:00
17 min

Name: Key

ID Number: ____________________

I pledge that I have neither given nor received any unauthorized assistance on this exam.

____
(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> allowed on all parts of the test, however you should still show all of your work.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
6. Make sure you sign the pledge and write your ID on both pages.
7. Number of questions = 8. Total Points = 70.

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Test 3

1. (10 points) Use calculus to find the absolute maximum and absolute minimum values of $g(t) = t^3 - 75t$ on the interval $[-3, 9]$.

absolute maximum is 198 when $t =$ -3 ✓

absolute minimum is -250 when $t =$ 5

✓ ✓✓ $g'(t) = 3t^2 - 75 = 3(t^2 - 25) = 3(t-5)(t+5)$ ✓

$g'(t) = 0$ if $t = 5$ and $t = -5$ ✓✓

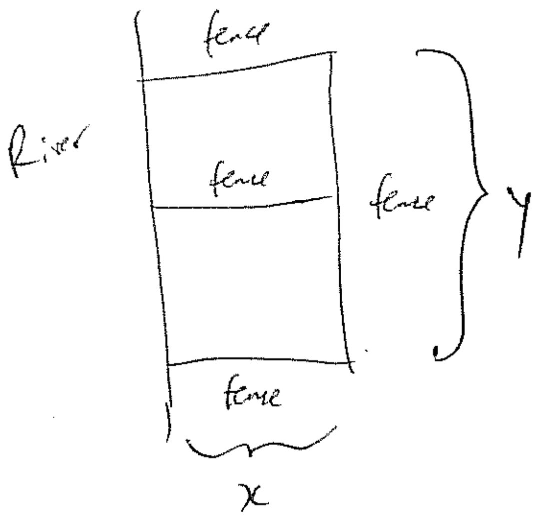
Check $g(-3) = 198 \Leftarrow$ abs max ✓

$g(5) = -250 \Leftarrow$ abs min ✓

$g(9) = 54$ ✓

-2 if max of 250

2. (10 points) Farmer Jane wants to build a rectangular pen to hold his llamas and goats. Since fencing is expensive, she is going to make one side of the pen be a river with a straight bank, so she only needs to build three sides of the rectangle. She also will put fencing down the middle of the pen to keep the llamas and goats apart (because we all know that llamas and goats don't get along). A diagram is below. If Jane wants the pen to have a total area of 1000 ft^2, find the *minimum* total amount of fencing she will need. Round your answer to the nearest 0.1 feet.



Total length of fence $= 3x + y$

$\text{Area} = xy$

$\Rightarrow 1000 = xy$

$y = \frac{1000}{x}$

$\Rightarrow L(x) = 3x + \frac{1000}{x}$

$L'(x) = 3 - \frac{1000}{x^2}$

$0 = 3 - \frac{1000}{x^2}$

$\frac{1000}{3} = x^2$

$x = 18.3$ feet. (18.257419)

$\Rightarrow \text{Total} = L(18.257) = \boxed{109.5 \text{ feet}}$

3. (5 points) Simplify the expression by using logarithm rules to combine the different terms:

$$3\ln(x) + \ln(4) - 2\ln(x^2+5) - \frac{1}{2}\ln(4x-1)$$

$= \ln x^3 + \ln 4 - \ln (x^2+5)^2 - \ln \sqrt{4x-1}$

$= \ln(4x^3) - (\ln (x^2+5)^2 \sqrt{4x-1})$

$= \boxed{\ln\left(\frac{4x^3}{(x^2+5)^2\sqrt{4x-1}}\right)}$

4. (7 points) The concentration of a drug in an organ at any time t (in seconds) is given by

$$x(t) = 0.08(1 - e^{-0.02t}) .$$

where $t \geq 0$ and $x(t)$ is measured in grams/cm^3. Round your answers to the nearest 0.0001.

(a) What will be the concentration of the drug after 20 seconds?

(b) When will the concentration be 0.05 grams/cm^3?

(c) What will be the concentration in the long run?

Explain your answer

(a) $x(20) = 0.0264$

(b) $0.05 = 0.08(1 - e^{-0.02t})$

$$\frac{0.05}{0.08} = 1 - e^{-0.02t}$$

$$e^{-0.02t} = 1 - \frac{5}{8}$$

$$t = \frac{\ln(1 - \frac{5}{8})}{-0.02}$$

$t = 49.0415$ seconds

(c) $\lim_{t \to \infty} x(t) = 0.08(1 - e^{-\infty})$

$= 0.08(1 - 0)$

$= \boxed{0.08}$

5. (12 points) Calculate the derivative of each function.

(a) $f(x) = (e^{x^3-2x})^3$

$$f'(x) = 3(e^{x^2-2x})^2(e^{x^2-2x})(2x-2)$$

$$= \boxed{3\left(e^{x^2-2x}\right)^3(2x-2)}$$

(b) $f(x) = \ln(7x^3 - 2x + 1)$

$$f'(x) = \frac{1}{7x^3-2x+1} \cdot (21x^2-2)$$

$$= \boxed{\frac{21x^2-2}{7x^3-2x+1}}$$

(c) $f(x) = \ln \dfrac{e^{(x^7)}(3x^2-x)}{x^5}$

$$f(x) = \ln e^{(x^2)} + \ln(3x^2-x) - \ln x^5$$

$$f(x) = x^2 + \ln(3x^2-x) - 5\ln x$$

$$f'(x) = \boxed{2x + \frac{6x-1}{3x^2-x} - \frac{5}{x}}$$

or $\ln(3x-1) - 4\ln x$

$\frac{3}{3x-1} - \frac{4}{x}$

or $2x + \frac{3}{3x-1} - \frac{4}{x}$

6. (6 points) Use logarithmic differentiation to calculate y' when $y = 5^x$.

$$\ln y = \ln 5^x$$

$$\frac{d}{dx}\left(\ln y = x(\ln 5) \right)$$

$$\frac{y'}{y} = \ln 5$$

$$\boxed{y' = (\ln 5) \cdot 5^x}$$

7. (12 points) Calculate the indefinite integrals.

(a) $\int \left(2e^x - 3 + \frac{4}{x}\right) dx \longrightarrow \frac{4}{x} = 4x^{-1}$

$= \boxed{2e^x - 3x + 4\ln|x| + C}$

(b) $\int x^3 \left(x^{10} + x^{-2} - \frac{1}{x^5}\right) dx$

$= \int (x^{13} + x - x^{-2})\, dx$

$= \frac{1}{14} x^{14} + \frac{x^2}{2} - (-1) x^{-1} + C$

$= \boxed{\frac{x^{14}}{14} + \frac{x^2}{2} + x^{-1} + C}$

8. (8 points) Phosphorus 32 (P-32) has a half-life of 13.4 days. If 100 g of this substance are present initially, find:

(a) An equation giving the amount present after t days.

$Q(t) = Q_0 e^{kt}$ ✓

$50 = 100 e^{k(13.4)}$ ✓

$\frac{\ln(\frac{1}{2})}{13.4} = \frac{13.4k}{13.4}$ ✓

$k = \frac{\ln(.5)}{13.4}$ ✓

$\Rightarrow Q(t) = 100 e^{\left(\frac{\ln(.5)}{13.4}\right)t}$ ✓

(b) The amount left after 9 days. Round your answer to the nearest 0.1 grams.

$Q(9) = 100 e^{\left(\frac{\ln(.5)}{13.4}\right) \cdot 9}$

$= \boxed{62.8 \text{ grams}}$

✓✓

Extra Credit(2 points) Calculate $f'(x)$ if $f(x) = \ln e^e$.

$f'(x) = 0$ b/c $\ln e^e$ is a constant.